Proceedings of the 6th Győr Symposium and 3rd Hungarian-Polish
and 1st Hungarian-Romanian Joint Conference on Computational Intelligence

Sensitivity Analysis of Fuzzy Signatures

I. Á. Harmati[1], Á. Bukovics[2], L. T. Kóczy[3]

[1]**Széchenyi István University, Dept. of Mathematics and Computational Sciences**
Egyetem tér 1, 9026, Győr, Hungary
E-mail: harmati@sze.hu
[2]**Széchenyi István University, Dept. of Structural and Geotechnical Engineering**
[3]**Széchenyi István University, Department of Automation**

Abstract: We briefly summarize our results on the sensitivity of the weighted generalized mean (power mean) aggregation operator and their application on the sensitivity analysis of fuzzy signatures.

Keywords: fuzzy signature, sensitivity, aggregation operator, weighted general mean, power mean

1. Introduction

Fuzzy signatures are hierarchical representations of data structuring into vectors of fuzzy values [1]. A fuzzy signature is defined as a special multidimensional fuzzy data structure, which is a generalization of vector valued fuzzy sets [2], [3], [4]. Vector valued fuzzy sets are special cases of L-fuzzy sets which were introduced in [5]. A fuzzy signature is denoted by

$$A\colon X \to S^{(n)},$$

where $1 \leq n$ and

$$S^{(n)} = \times_{i=1}^{n} S_i \qquad S_i = \begin{cases} [0,1] \\ S^{(m)} \end{cases}$$

We can represent a fuzzy signature by nested vector value fuzzy sets and also by a tree graph (see Figure 1), which is much more understandable [6].

The goal of this article is to discuss how the membership value of the whole fuzzy set changes if the membership values in the nested vectors change. In other words, if we think of the tree graph representation, how the membership value of the root changes if the membership values of leaves change. To answer this question we have to know how to compute a membership value of a subgraph from the leaves [7].

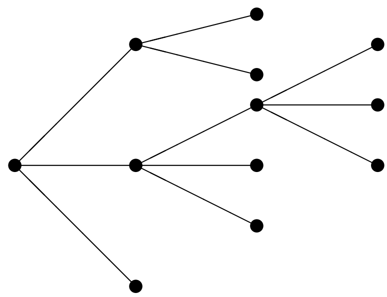

Figure 1. A fuzzy signature graph.

2. Sensitivity of the WGM aggregation operator

The generalized mean and its generalization, the weighted generalized mean (WGM) form a very large class of aggregation operators. Their various special cases often arise also in theoretical and practical problems. In the following we assume that all the operators applied on the membership values in the signature are from the class of the weighted generalized mean aggregation operators.

Definition. Let $x_1, \ldots, x_n$ and $w_1, \ldots, w_n$ be nonnegative real numbers, $w_i \geq 0, \sum_{i=1}^{n} w_i = 1$ and $p \in \mathbb{R}$ ($p \neq 0$). Then the weighted generalized mean (WGM) of $x_1, \ldots, x_n$ with weights $w_1, \ldots, w_n$ and with parameter p:

$$M_p^w(x_1, \ldots, x_n) = \left[\sum_{k=1}^{n} w_k x_k^p \right]^{\frac{1}{p}}$$

The limits at $\pm\infty$ regardless to the weights:

$$\lim_{p \to \infty} \left[\sum_{k=1}^{n} w_k x_k^p \right]^{\frac{1}{p}} = \max(x_i) \qquad \lim_{p \to -\infty} \left[\sum_{k=1}^{n} w_k x_k^p \right]^{\frac{1}{p}} = \min(x_i)$$

The limit if $p \to 0$ is the weighted geometric mean:

$$\lim_{p\to 0}\left[\sum_{k=1}^{n} w_k x_k^p\right]^{\frac{1}{p}} = \prod_{i=1}^{n} x_i^{w_i}$$

What can be said about the change of M ($|\Delta M|$) if we know the change of the input? We give upper bound on $|\Delta M|$ if we know the norm of the change of the input vector ($\|\Delta \underline{x}\|$). Now we restrict the results to the most widely used norms: sum of absolute values ($\|\cdot\|_1$), euclidean ($\|\cdot\|_2$) and maximum ($\|\cdot\|_\infty$) norms (the general case for arbitrary p'-norm is also solved).

Table 1. Values of the coefficient K_1 for $|\Delta M| \le K_1 \cdot \|\Delta \underline{x}\|_1$.

value of p	K_1	if $w_i = 1/n$
$p < 0$	$\max\left\{w_i^{1/p}\right\} = \|\underline{w}^{1/p}\|_\infty$	$n^{-1/p}$
$0 \le p < 1$	-	-
$p \ge 1$	$\max\left\{w_i^{1/p}\right\} = \|\underline{w}^{1/p}\|_\infty$	$n^{-1/p}$

Table 2. Values of the coefficient K_2 for $|\Delta M| \le K_2 \cdot \|\Delta \underline{x}\|_2$.

value of p	K_2	if $w_i = 1/n$
$p < 0$	$\max\left\{w_i^{1/p}\right\} = \|\underline{w}^{1/p}\|_\infty$	$n^{-1/p}$
$0 \le p < 1$	-	-
$p = 1$	$\|\underline{w}\|_2$	$n^{-1/2}$
$1 < p < 1.5$	$\max\left\{w_i\right\}^{1/2} = \|\underline{w}^{1/2}\|_\infty$	$n^{-1/2}$
$1.5 \le p < 2$	$\max\left\{w_i^{1/p}\right\} \cdot n^{1/p-1/2}$	$n^{-1/2}$
$2 \le p$	$\max\left\{w_i^{1/p}\right\} = \|\underline{w}^{1/p}\|_\infty$	$n^{-1/p}$

Table 3. Values of the coefficient K_∞ for $|\Delta M| \leq K_\infty \cdot \|\Delta \underline{x}\|_\infty$.

value of p	K_∞	if $w_i = 1/n$
$p < 0$	$\max\left\{w_i^{1/p}\right\} = \|\underline{w}^{1/p}\|_\infty$	$n^{-1/p}$
$0 \leq p < 1$	-	-
$1 \leq p < 2$	1	1
$2 \leq p$	$\max\left\{w_i^{1/p}\right\} \cdot n^{1/p} = \|\underline{w}^{1/p}\|_\infty \cdot n^{1/p}$	1

3. Sensitivity of fuzzy signatures

Based on the results above we analyse the sensitivity of fuzzy signatures in which the values are determined by a WGM operators in every nodes. The sensitivity bound of the whole fuzzy signature is derived from the bounds of the WGM-s, according to the graph structure of the signature. The output (the membership value of the signature) is denoted by f.

- In $\|\cdot\|_1$ norm of the input vector:
  Let us denote by K_{11} the bound for the WGM applied in the root of the signature and by $\Delta\underline{x}_{11}$ of the change of its input vector; the bounds for their WGM operators are $K_{21}, \ldots, K_{2n_2}$ (n_2 is the number of vertices to the root), the change of their inputs are $\Delta\underline{x}_{21}, \ldots, \Delta\underline{x}_{2n_2}$ etc., up to the end of the graph. Then change of the output value can be estimated by the following way:

$$
\begin{aligned}
|\Delta f| &\leq K_{11} \cdot \|\Delta\underline{x}_{11}\|_1 \leq\leq K_{11} \cdot \big(K_{21} \cdot \|\Delta\underline{x}_{21}\|_1 + \cdots + K_{2n_2} \cdot \|\Delta\underline{x}_{2n_2}\|_1\big) \\
&\vdots \\
&\leq \sum_{i=1}^{N} K_i \cdot |\Delta x_i| \leq \max(K_i) \cdot \sum_{i=1}^{N} |\Delta x_i| = \max(K_i) \cdot \|\Delta\underline{x}\|_1
\end{aligned}
$$

  where K_i is the product of the bounds form the root to the i-th leaf.

- In $\|\cdot\|_2$ norm of the input vector:
  Now it is more convenient to deal with $|\Delta f|^2$ instead of $|\Delta f|$. The estimation works quite similar as in the previous case. The C_{**}-s denote the squares of the bounds for

the WGM operators. The estimation:

$$
\begin{aligned}
|\Delta f|^2 &\le C_{11}^2 \cdot \|\Delta \underline{x}_{11}\|_2^2 \le C_{11}^2 \cdot \left(C_{21}^2 \cdot \|\Delta \underline{x}_{21}\|_2^2 + \cdots + C_{2n_2}^2 \cdot \|\Delta \underline{x}_{2n_2}\|_2^2\right) \\
&\vdots \\
&\le \sum_{i=1}^{N} C_i^2 \cdot |\Delta x_i|^2 \le \max(C_i^2) \cdot \sum_{i=1}^{N} |\Delta x_i|^2 = \max(C_i^2) \cdot \|\Delta \underline{x}\|_2^2
\end{aligned}
$$

where C_i^2 is the product of the squares of the bounds from the root to the i-th leaf.

- In $\|\cdot\|_\infty$ norm of the input vector:
  This case differs a bit form the others because of the $\max$ operator. The D_{**}-s are the bounds for the WGM operators.

$$
\begin{aligned}
|\Delta f| &\le D_{11} \cdot \|\Delta \underline{x}_{11}\|_\infty \le D_{11} \cdot \max\left(D_{21} \cdot \|\Delta \underline{x}_{21}\|_\infty, \ldots, D_{2n_2} \cdot \|\Delta \underline{x}_{2n_2}\|_\infty\right) \\
&\vdots \\
&\le \max(D_i) \cdot \|\Delta \underline{x}\|_\infty
\end{aligned}
$$

where D_i is the product of the greatest bounds at every level.

The sensitivity analysis of a fuzzy signature becomes much more simple if the fuzzy signature is homogeneous, i.e. if all of the aggregation operators in the nodes are weighted generalized mean operators with the same value of p. If this condition holds, the output vale of the signature is the weighted generalized mean of the input values with parameter p, where the weights are the product of the weights form the root to the leaves.

As a practical example, the sensitivity analysis of a fuzzy signature which was applied for status-determining and ranking buildings was also discussed in [7].

Acknowledgement

This work was supported by TÁMOP-4.2.2.A-11/1/KONV-2012-0012, and by the Hungarian Scientific Research Fund (OTKA) K105529 and K108405. This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4.A/2-11-1-2012-0001 'National Excellence Program'.

References

[1] C. Pozna, N. Minculete, R. E. Precup, L. T. Kóczy, Á. Ballagi, *Signatures: Definitions, operators and applications to fuzzy modelling*, Fuzzy Sets and Systems 201 (2012), pp. 86-104.

[2] T. Vámos, L. T. Kóczy , G. Biró, *Fuzzy signatures in datamining*, in: Proceedings of the Joint 9th IFSA World Congress and 20th NAFIPS International Conference, Vancouver, BC, Canada, 2001, pp. 2842-2846 (5).

[3] K. W. Wong, T. D. Gedeon, L. T. Kóczy, *Construction of fuzzy signature from data: an example of SARS pre-clinical diagnosis system*, in: Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE2004), Budapest, Hungary, 2004, pp.1649-1654.

[4] Á. Ballagi, L. T. Kóczy, T. D. Gedeon, *Robot cooperation without explicit communication by fuzzy signatures and decision trees*, in: Proceedings of theJoint 2009 International Fuzzy Systems Association World Congress and 2009 European Society of Fuzzy Logic and Technology Conference (IFSA-EUSFLAT2009), Lisbon, Portugal, 2009, pp.1468-1473.

[5] J.A.Goguen, *L-fuzzysets*, J.Math.Anal.Appl.18(1) (1967), pp. 145-174.

[6] L. T. Kóczy, T. Vámos, G. Biró, *Fuzzy signatures*, in: Proceedings of the 4th Meeting of the Euro Working Group on Fuzzy Sets and the 2nd International Conference on Soft and Intelligent Computing (EUROPUSE-SIC'99), Budapest, Hungary, 1999, pp.210-217.

[7] I. Á. Harmati, Á. Bukovics, L. T. Kóczy *Sensitivity Analysis of the Weighted Generalized Mean Aggregation Operator and its Application to Fuzzy Signatures*, in: Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2014), Beijing, China, 2014.