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Using Signatures for Expert System Modeling Case studies

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The new modeling approach presented in the previous paper is formulated as an original three-step algorithm that maps the signatures onto expert systems. The algorithm has two inputs represented by the knowledge base (the rules) and the data base (the facts). The algorithm constructs the signatures which represent expert system models. This paper is organized as follows: a short overview on signatures and on their operators is presented in the next Section. Section 1 validates the theoretical approach by two case studies focused on the construction of models of a deterministic and of a Bayesian expert system. The conclusions are outlined in Section 5.

1. Case Studies

The expert system modeling algorithm proposed in the previous paper is exemplified here by two case studies which produce models of a deterministic and of a Bayesian rule-based expert system. The new models are expressed as signatures defined in Section 2.

1.1. Case Study 1: Signature-Based Modeling of a Deterministic Expert System

This case study is focused on the "media advisor" rule-based expert system taken from [26] which provides advice on selecting a medium for delivering a training program based on the trainee's job. Starting with the rules of this system, our expert system modeling algorithm is applied as follows to model the inference engine by means of signatures. The rules, their symbols and the signatures and are presented in Table 1.

Table 1 Rules, symbols and signatures in case study 1

Nr.	Rule	Symbols	Signature
1	If [The environment is paper] or [The environment is manuals] or [The environment is documents] or [The environment is textbooks] Then [The stimulus_situation is verbal]	[The environment is paper] = E_1 [The environment is manuals]=E_2 [The environment is documents]=E_3 [The environment is textbooks]=E_4 [The stimulus_situation is verbal]=Ss_1	$r_1 = [E_1, E_2, E_3, E_4]^T$, ${}^g@(r_1) = [Ss_1]^T$
2	If [The environment is pictures] or [The environment is illustrations] or [The environment is photographs] or [The environment is diagrams] Then [The stimulus_situation is visual]	[The environment is pictures]= E_5 [The environment is illustrations]= E_6 [The environment is photographs]= E_7 [The environment is diagrams]= E_8 [The stimulus_situation is visual]= Ss_2	$r_2 = [E_5, E_6, E_7, E_8]^T$, ${}^g@(r_2) = [Ss_2]^T$
3	If [The environment is machines] or [The environment is buildings] or [The environment is tools] Then [The stimulus_ situation is 'physical object']	[The environment is machines] = E_9 [The environment is buildings] = E_{10} [The environment is tools] = E_{11} [The stimulus_ situation is 'physical object'] = Ss_3	$r_3 = [E_9, E_{10}, E_{11}]^T$, ${}^g@(r_3) = [Ss_3]^T$
4	If [The environment is numbers] or [The environment is formulas] or [The environment is computer program] Then [The stimulus_ situation is symbolic]	[The environment is numbers] = E_{12} [The environment is formulas] = E_{13} [The environment is computer program] = E_{14} [The stimulus_ situation is symbolic] = Ss_4	$r_4 = [E_{12}, E_{13}, E_{14}]^T$, ${}^g@(r_4) = [Ss_4]^T$
5	If [The job is lecturing] or [The job is advising] or [The job is counseling]	[The job is lecturing] = J_1 [The job is advising] = J_2 [The job is counseling] = J_3 [The stimulus_ response is	$r_5 = [J_1, J_2, J_3]^T$, ${}^g@(r_5) = [Sr_1]^T$

	Then [The stimulus_ response is oral]	oral]= Sr_1	
6	If [The job is building] or [The job is repairing] or [The job is troubleshooting] Then [The stimulus_ response is 'hands-on']	[The job is building] = J_4 [The job is repairing] = J_5 [The job is troubleshooting] = J_6 [The stimulus_ response is 'hands-on']= Sr_2	$r_6 = [J_4, J_5, J_6]^T$, ${}^g@(r_6) = [Sr_2]^T$
7	If [The job is writing] or [The job is typing] or [The job is drawing] Then [The stimulus_ response is document]	[The job is writing] = J_7 [The job is typing] = J_8 [The job is investigating] = J_9 [The stimulus_ response is document] = Sr_3	$r_7 = [J_7, J_8, J_9]^T$, ${}^g@(r_7) = [Sr_3]^T$
8	If [The job is evaluating] or [The job is reasoning] or [The job is investigating] Then [The stimulus_ response is analytical]	[The job is evaluating]= J_{10} [The job is reasoning] = J_{11} [The job is investigating] = J_{12} [The stimulus_ response is analytical] = Sr_4	$r_8 = [J_{10}, J_{11}, J_{12}]^T$, ${}^g@(r_8) = [Sr_4]^T$
9	If [The stimulus_ situation is 'physical object'] and [The stimulus_ response is 'hands-on'] and [The feedback is required] Then [The medium is workshop]	[The stimulus_ situation is 'physical object'] = Ss_3 [The stimulus_ response is 'hands-on'] = Sr_2 [The feedback is required] = F_1 [The medium is workshop] = M_1	$r_9 = [Ss_3, Sr_2, F_1]^T$, ${}^f@(r_9) = [M_1]^T$
10	If [The stimulus_ situation is symbolic] and [The stimulus_ response is analytical] and [The feedback is required] Then [The medium is lecture_ tutorial]	[The stimulus_ situation is symbolic] = Ss_4 [The stimulus_ response is analytical] = Sr_4 [The feedback is required] = F_1 [The medium is lecture_ tutorial] = M_2	$r_{10} = [Ss_4, Sr_4, F_1]^T$, ${}^f@(r_{10}) = [M_2]^T$
11	If [The stimulus_ situation is visual] and [The stimulus_ response is documented] and [The feedback is	[The stimulus_ situation is visual] = Ss_2 [The stimulus_ response is documented] = Sr_3 [The feedback is not required] = F_2	$r_{11} = [Ss_2, Sr_3, F_2]^T$, ${}^f@(r_{11}) = [M_3]^T$

	not required] Then [The medium is videocassette]	[The medium is videocassette] =M_3	
12	If [The stimulus_ situation is visual] and [The stimulus_ response is oral] and [The feedback is required] Then [The medium is lecture_ tutorial]	[The stimulus_ situation is visual] = Ss_2 [The stimulus_ response is oral] = Sr_1 [The feedback is required] = F_1 [The medium is lecture_ tutorial] = M_2	$r_{12} = [Ss_2, Sr_1, F_1]^T$, ${}^f@(r_{12}) = [M_2]^T$
13	If [The stimulus_ situation is verbal] and [The stimulus_ response is analytical] and [The feedback is required] Then [The medium is lecture_ tutorial]	[The stimulus_ situation is verbal] = Ss_1 [The stimulus_ response is analytical] = Sr_4 [The feedback is required] = F_1 [The medium is lecture_ tutorial] = M_2	$r_{13} = [Ss_1, Sr_4, F_1]^T$, ${}^f@(r_{13}) = [M_2]^T$
14	If [The stimulus_ situation is verbal] and [The stimulus_ response is oral] and [The feedback is required] Then [The medium is role play exercises]	[The stimulus_ situation is verbal] = Ss_1 [The stimulus_ response is oral] = Sr_1 [The feedback is required] = F_1 [The medium is role play exercises] = M_4	$r_{14} = [Ss_1, Sr_1, F_1]^T$, ${}^f@(r_{14}) = [M_4]^T$

We will compute the expert system output for the following observed facts:

- the environment is machines: E_9,
- the job is repairing: J_5,
- the feedback is required: F_1.

The three steps of our algorithm are applied as follows.

Step 1. The signature is

$$r = [M_1, M_2, M_3, M_4]^T, \tag{42}$$

and the algorithm memorizes

$${}^g@(r) = [M]^T, \tag{43}$$

$$g:\{0,1\}^4 \to \{M_1,...,M_4,\Phi\},\ g(x_1,...,x_4)=\begin{cases} M_i, & \text{if} \quad x_i=1,\ x_j=0\ \forall\ j\neq i \\ \Phi, & \text{if} \quad x_i=0,\ i=1...4 \\ F_i, & \text{if} \quad x_i=1,\ x_j=1,\ p(i)>p(j)\ \forall\ j\neq i \end{cases} \tag{44}$$

where $p(i)$ is the priority of rule r_i, and Φ indicates no rules to apply, in accordance with the previous section.

Step 2. The first iteration leads to the initial signature

$$\begin{aligned} r &= \overline{@}_2^3(r)\oplus_1 r_9 \oplus_{2,1} r_{10} \oplus_{2,2} r_{12} \oplus_{2,3} r_{13} \oplus_3 r_{11} \oplus_3 r_{14} \\ &= [[Ss_3,Sr_2,F_1],[[Ss_4,Sr_4,F_1],[Ss_2,Sr_1,F_1],[Ss_1,Sr_4,F_1]],[Ss_2,Sr_3,F_2],[Ss_1,Sr_1,F_1]]^T, \end{aligned} \tag{45}$$

and the algorithm memorizes

$$\begin{aligned} &{}^f@_1(r)=[M_1,[[Ss_4,Sr_4,F_1],[Ss_2,Sr_1,F_1],[Ss_1,Sr_4,F_1]],[Ss_2,Sr_3,F_2],[Ss_1,Sr_1,F_1]]^T, \\ &{}^g@_2(r)=[[Ss_3,Sr_2,F_1],M_2,[Ss_2,Sr_3,F_2],[Ss_1,Sr_1,F_1]]^T, \\ &{}^f@_{2,1...3}(r)=[[Ss_3,Sr_2,F_1],[M_2,M_2,M_2],[Ss_2,Sr_3,F_2],[Ss_1,Sr_1,F_1]]^T, \\ &{}^f@_3(r)=[[Ss_3,Sr_2,F_1],[[Ss_4,Sr_4,F_1],[Ss_2,Sr_1,F_1],[Ss_1,Sr_4,F_1]],M_3,[Ss_1,Sr_1,F_1]]^T, \\ &{}^f@_4(r)=[[Ss_3,Sr_2,F_1],[[Ss_4,Sr_4,F_1],[Ss_2,Sr_1,F_1],[Ss_1,Sr_4,F_1]],[Ss_2,Sr_3,F_2],M_4]^T. \end{aligned} \tag{46}$$

where *f* is defined in (27).

All facts involved in *r* are inferable along with F_1 and F_2. F_1 is observed and it can be replaced with 1, but F_2 is not observed. This means that the corresponding branch must be pruned:

$$r=\varnothing_3(r)=[[Ss_3,Sr_2,1],[[Ss_4,Sr_4,1],[Ss_2,Sr_1,1],[Ss_1,Sr_4,1]],[Ss_1,Sr_1,1]]^T. \tag{47}$$

This iteration ends with the simplification

$$\begin{aligned} &{}^f@_1(r)=[M_1,[[Ss_4,Sr_4,1],[Ss_2,Sr_1,1],[Ss_1,Sr_4,1]],[Ss_1,Sr_1,1]]^T, \\ &{}^g@_2(r)=[[Ss_3,Sr_2,1],M_2,[Ss_1,Sr_1,1]]^T, \\ &{}^f@_{2,1...3}(r)=[[Ss_3,Sr_2,1],[M_2,M_2,M_2],[Ss_1,Sr_1,1]]^T, \\ &{}^f@_3(r)=[[Ss_3,Sr_2,1],[[Ss_4,Sr_4,1],[Ss_2,Sr_1,1],[Ss_1,Sr_4,1]],M_4]^T. \end{aligned} \tag{48}$$

The second iteration leads to the initial signature

$$r = r \oplus_{1,1} r_3 \oplus_{1,2} r_6 \oplus_{2,1,1} r_1 \oplus_{2,1,2} r_8 \oplus_{2,2,1} r_4 \oplus_{2,2,2} r_8 \oplus_{2,3,1} r_2 \oplus_{2,3,2} r_5 \oplus_{3,1} r_1 \oplus_{3,2} r_5$$
$$= [[[E_9, E_{10}, E_{11}],[J_4, J_5, J_6],1],$$
$$[[[[E_{12}, E_{13}, E_{14}],[J_{10}, J_{11}, J_{12}],1],[[E_5, E_6, E_7, E_8],[J_1, J_2, J_3],1],[E_1, E_2, E_3, E_4],[J_{10}, J_{11}, J_{12}],1]],$$
$$[[E_1, E_2, E_3, E_4],[J_1, J_2, J_3],1]]^T, \tag{49}$$

and the algorithm memorizes

$$^f@_{1,1}(r) = [Ss_3,[J_4, J_5, J_6],1],$$
$$[[[E_{12}, E_{13}, E_{14}],[J_{10}, J_{11}, J_{12}],1],[[E_5, E_6, E_7, E_8],[J_1, J_2, J_3],1],[[E_1, E_2, E_3, E_4],[J_{10}, J_{11}, J_{12}],1]],$$
$$[[E_1, E_2, E_3, E_4],[J_1, J_2, J_3],1]]^T,$$
$$^f@_{1,2}(r) = [[[E_9, E_{10}, E_{11}], Sr_2,1],$$
$$[[[E_{12}, E_{13}, E_{14}],[J_{10}, J_{11}, J_{12}],1],[[E_5, E_6, E_7, E_8],[J_1, J_2, J_3],1],[[E_1, E_2, E_3, E_4],[J_{10}, J_{11}, J_{12}],1]],$$
$$[[E_1, E_2, E_3, E_4],[J_1, J_2, J_3],1]]^T,$$
$$\ldots,$$
$$^f@_{3,2}(r) = [[[E_9, E_{10}, E_{11}],[J_4, J_5, J_6],1],$$
$$[[[E_{12}, E_{13}, E_{14}],[J_{10}, J_{11}, J_{12}],1],[[E_5, E_6, E_7, E_8],,1],[[E_1, E_2, E_3, E_4],[J_{10}, J_{11}, J_{12}],1]],$$
$$[[E_1, E_2, E_3, E_4], Sr_1,1]]^T. \tag{50}$$

All facts from this signature are observable but not inferable. All facts involved in r are not observed along with E_9 and J_5.

Step 3. The following signature is produced:

$$r = [[[1,0,0],[0,1,0],1],[[[0,0,0,0],[0,0,0],1],[[0,0,0],[0,0,0],1],[[0,0,0,0],[0,0,0],1]],$$
$$[[0,0,0,0],[0,0,0],1]]^T. \tag{51}$$

The expert system output can be computed by successive contractions:

$$r={}^f@_{1,1}\,{}^f@_{1,2}\,{}^f@_{2,1,1}\,{}^f@_{2,1,2}\,{}^f@_{2,2,1}\,{}^f@_{2,2,2}\,{}^f@_{3,1}\,{}^f@_{3,2}(r) = [[1,1,1],[[0,0,1],[0,0,1],[0,0,1]],[0,0,1]]^T,$$
$$r={}^f@_1\,{}^f@_{2,1}\,{}^f@_{2,2}\,{}^f@_{2,3}\,{}^f@_3(r) = [1,[0,0,0],0]^T,$$
$$r={}^g@_2(r) = [1,0,0]^T,$$
$$r={}^g@(r) = [M_1]^T, \tag{52}$$

which employ the contraction law (44).

This result means that the response of the expert system output is characterized by the expert system output “The medium is workshop”.

1.2. Case Study 2: Signature-Based Modeling of a Bayesian Expert System

This case study considers the signature-based modeling of an uncertain rule-based expert system represented by a Bayesian expert system. This rule-based expert system is taken from [26], and it predicts the "tomorrow" weather, i.e., tomorrow will rain or not. It is an accumulation of evidence expert system whose signification is that posterior probability is inherited from one rule to another one. Two signatures will be constructed as follows using the algorithm defined in Section 3, the first one will ignore the accumulation of evidence and the second will consider the accumulation of evidence.

Prior to the presentation of the application of our expert system modeling algorithm, some details on the firing of Bayesian rules are given as follows in relation with the rule

$$\text{Rule}\,1: \textit{If}\ \ A\,\{\text{LS}\,x, \text{LN}\,y\}\ \textit{Then}\ B\,\{\text{prior}\,u\}, \tag{54}$$

where $\text{LS} \equiv x$ is the likelihood of sufficiency of fact *A*, $\text{LN} \equiv y$ is the likelihood of necessity of fact *A*, and *u* is the prior probability of fact *B*. The application of Definition 8 leads to

$$r_1 = [A]^T,\ \ {}^{h}@(r_1) = [B]^T, \tag{55}$$

where the definition of the function *h* is

$$h(A) = \begin{cases} \dfrac{O(B \mid A)}{1 + O(B \mid A)} & if \quad A \\ \dfrac{O(B \mid \neg A)}{1 + O(B \mid \neg A)} & if \quad \neg A \end{cases},\ \ \begin{cases} O(B \mid A) = x\,O(B) \\ O(B \mid \neg A) = y\,O(B) \end{cases},\ \ O(B) = \frac{u}{1-u}, \tag{56}$$

$O(B)$ is the prior evidence of fact *B*, $O(B \mid A)$ is the posterior evidence of fact *B* given the fact *A* (true); *and*; $h(A)$ is the posterior probability of fact *B* given the fact *A*.

The rules and signatures of the expert system considered in the case study 2 are synthesized in Table 2, which is similar to Table 1, *f* is defined in (26), and the general notation $h \circ f = h(f)$ is used.

Table 2 Rules, symbols and signatures in case study 2

Nr.	Rule	Symbol	Signature
1	If [Today is rain] {LS=2.5; LN=0.6} Then [Tomorrow is rain] {prior=0.5}	[Today is rain] = *TyR* [Tomorrow is rain]=*TwR*	$r_1 = [TyR]^T$, ${}^{h_1 \circ f}@(r_1) = [TwR]^T$
2	If [Today is dray] {LS=1.6; LN=0.4} Then [Tomorrow is dray] {prior=0.5}	[Today is dray] = *TyD* [Tomorrow is dray] = *TwD*	$r_2 = [TyD]^T$, ${}^{h_2 \circ f}@(r_2) = [TwD]^T$

3	If [Today is rain] and [Rainfall is low]{LS=10; LN=1} Then [Tomorrow is dray] {prior=0.5}	[Today is rain] = *TyR* [Rainfall is low] = *RaL* [Tomorrow is dray] = *TwD*	$r_3 = [TyR, RaL]^T$, ${}^{h_3 \circ f}@(r_3) = [TwD]^T$
4	If [Today is rain] and [Rainfall is low] and [Temperature is cold] {LS=1.5; LN=1} Then [Tomorrow is dray] {prior=0.5}	[Today is rain] = *TyR* [Rainfall is low] = *RaL* [Temperature is cold] = *TeC* [Tomorrow is dray] = *TwD*	$r_4 = [TyR, RaL, TeC]^T$, ${}^{h_3 \circ f}@(r_3) = [TwD]^T$
5	If [Today is dray] and [Temperature is warm]{LS=2; LN=0.9} Then [Tomorrow is rain] {prior=0.5}	[Today is dray] = *TyD* [Temperature is warm] = *TeW* [Tomorrow is rain]= *TwR*	$r_5 = [TyD, TeW]^T$, ${}^{h_5 \circ f}@(r_5) = [TwR]^T$
6	If [Today is dray] and [Temperature is warm] and [sky is overcast]{LS=5; LN=1} Then [Tomorrow is rain] {prior=0.5}	[Today is dray] = *TyD* [Temperature is warm] = *TeW* [Sky is overcast] = *SyO* [Tomorrow is rain] = *TwR*	$r_6 = [TyD, TeW, SyO]^T$, ${}^{h_6 \circ f}@(r_6) = [TwR]^T$

If the accumulation of evidence is ignored the application of the three steps of our expert system modeling algorithm are first presented as follows.

Step 1. The signature is

$$r = [TwD, TwR]^T, \tag{57}$$

and the algorithm memorizes

$${}^{g}@(r) = [Tw]^T. \tag{58}$$

where $Tw = g(TwD, TwR) = \begin{cases} TwD, & \text{if} \quad TwD > TwR, \\ TwR, & \text{if} \quad TwR > TwD. \end{cases}$

Step 2. The signature is

$$r = \overline{@}_2^3(\overline{@}_1^3(r)) = [[TwD_2, TwD_3, TwD_4], [TwR_1, TwR_5, TwR_6]]^T , \qquad (59)$$

and the algorithm memorizes

$$\begin{aligned} &{}^g @_1(r) = [TwD, [TwR_1, TwR_5, TwR_6]]^T (r) = [Tw]^T , \\ &{}^g @_2(r) = [[TwD_2, TwD_3, TwD_4], TwR]^T . \end{aligned} \qquad (60)$$

Step 3. The results are:

$$\begin{aligned} &r = r \oplus_{1,1} r_2 \oplus_{1,2} r_3 \oplus_{1,3} r_4 \oplus_{2,1} r_1 \oplus_{2,2} r_5 \oplus_{2,3} r_6 \\ &= [[[TyD], [TyR, RaL], [TyR, RaL, TeC]], [[TyR], [TyD, TeW], [TyD, TeW, SyO]]]^T , \\ &{}^{h_2 \circ f} @_{1,1}(r) = [[TwD_2, [TyR, RaL], [TyR, RaL, TeC]], [[TyR], [TyD, TeW], [TyD, TeW, SyO]]]^T , \\ &{}^{h_3 \circ f} @_{1,2}(r) = [[[TyD], TwD_3, [TyR, RaL, TeC]], [[TyR], [TyD, TeW], [TyD, TeW, SyO]]]^T , \\ &\ldots, \\ &{}^{h_6 \circ f} @_{2,3}(r) = [[[TyD], [TyR, RaL], [TyR, RaL, TeC]], [[TyR], [TyD, TeW], TwR_6]]^T . \end{aligned} \qquad (61)$$

For the observations [Today is rain], [Rainfall is low], [Temperature is cold], [Sky is overcast] (TyR=1, RaL=1, TeC=1, SyO=1), the expert system output is computed in terms of:

$$\begin{aligned} &r = [[[TyD], [TyR, RaL], [TyR, RaL, TeC]], [[TyR], [TyD, TeW], [TyD, TeW, SyO]]]^T \\ &= [[[0], [1,1], [1,1,1]], [[1], [0,0], [0,0,1]]]^T , \\ &r = {}^f @_{1,1}({}^f @_{1,2}({}^f @_{1,3}({}^f @_{2,1}({}^f @_{2,2}({}^f @_{2,3}(r)))))) = [[[0], [1], [1]], [[1], [0], [0]]]^T , \\ &r = {}^{h_2} @_{1,1}({}^{h_3} @_{1,2}({}^{h_4} @_{1,3}({}^{h_1} @_{2,1}({}^{h_5} @_{2,2}({}^{h_6} @_{2,3}(r)))))) = [[0.285, 0.909, 0.6], [0.714, 0.473, 0.5]]^T , \\ &r = {}^g @_1({}^g @_2(r)) = [0.909, 0.714]^T . \end{aligned} \qquad (62)$$

This expert system response shows that the probability for “Tomorrow is Dray” is 0.909 and the probability for “Tomorrow is Rain” is 0.714. Prior to observations, the probabilities have been equal, i.e., 0.5 to 0.5.

If the accumulation of evidence is not ignored, the following signatures are obtained in this case study:

$$r = r_1 = [TyR]^T,$$
$$r = \Theta_2(r) \oplus_2 r_2 = [[TyR],[TyD]]^T,$$
$$...,$$
$$r = \Theta_2(r) \oplus_2 r_6 = [[[[[[TyR],[TyD]],[TyR,RaL]],[TyR,RaL,TeC]],[TyD,TeW]],[TyD,TeW,SyO]]^T,$$
$$r = [[[[[[TyR],[TyD]],[TyR,RaL]],[TyR,RaL,TeC]],[TyD,TeW]],[TyD,TeW,SyO]]^T. \tag{63}$$

For the mentioned observations (*TyR*=1, *RaL*=1, *TeC*=1, *SyO*=1), the constructed signature is

$$r = [[[[[[1],[0]],[1,1]],[1,1,1]],[0,0]],[0,0,1]]^T. \tag{64}$$

The firing is computed in this case using the functions *h* in six iterations. The difference with respect to the previous case concerns a new iteration which assumes the previous iteration result expressed as the prior probability. Therefore the expert system response is

$$r = {}^{h_6}@({}^{h_5}@_1({}^{h_4}@_{1,1}({}^{h_3}@_{1,1,1}({}^{h_2}@_{1,1,1,1}({}^{h_1}@_{1,1,1,1,1}(r)))))) = [0.86, 0.69]^T. \tag{65}$$

This result indicates that the probability for “Tomorrow is Dray” is 0.86, and the probability for “Tomorrow is Rain” is 0.69.

5. Conclusions

Two case studies have been presented with this regard with focus on deterministic and nondeterministic rule-based expert systems. The differences concern the definitions of the functions f and g.

The drawback of the approach presented in this paper is the need to express the signatures. However, as shown in [29], this process can be carried out in an easily algorithmic manner by the implementation of the operators on signatures as software objects.

The future research will be focused on the reduction of the number of iterations which correspond to the step 2 of the algorithm. Optimization approaches can be used with this regard [3, 15, 27, 33, 35]. More convincing applications including fuzzy logic control ones will be tackled.

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