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Cost of reaching the consensus in a group of agents based on a psychology of opinion change

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Abstract: We deal with consensus reaching, and its related decision support system. We assume that consensus reaching proceeds in a small group of agents who shows their preferences with respect to a set of options. We propose an approach based on the cost of reaching the consensus in the sense of evaluation of preference updating, based on psychological and sociological reasons. We use a model with nonlinear functions representing human behaviour over the interaction of some endogenous and exogenous forces.

Keywords: *group decision making, consensus, psychology of opinion change, cost function*

1. Introduction

Our research deals with supporting reaching a consensual solution in a group decision making problems [6],[7]. Essentially, we assume a small set of agents who openly express their opinions as to some set of options. Agents' testimonies strongly disagree in the early stage of the process and the main goal is to get them close enough to the consensus, i.e. to the highest acceptable consent as to the mutually agreed decision. For the sake of consensus, agents are expected to modify their preferences in the direction indicated by the moderator, a person who measures the agreement within the group and provides appropriate changes in preference relations aimed at increasing the level of consensus. What matters here is that it takes effort for the moderator to lead agents towards the mutual agreement and instruct them how to update their testimonies in order to increase the level of consensus.

In this paper we attempt to measure the cost of consensus reaching process by the function corresponding to changes in agents' preferences in a small group of people [1]. We use social and psychological terminology and a mathematical formalism in order to model a human behaviour over the interaction of some endogenous and exogenous

forces. We distinguish either a nonlinear force which tries to bring back the current agent's opinion towards his initial testimony or a nonlinear influence of the moderator who brings opinions closer to each other but becoming weaker for high opinion differences. We use some assumptions from the model presented by Gabbay in [3],[4].

The paper is organized as follows. Psychological backgrounds of the cost based scheme for the evaluation of preference updating are provided in Section 2. Section 3 presents mathematical formalism of the proposed model. Conclusion and some directions of further discussion are contained in Section 4.

2. Dual cost based scheme – psychological reasons

In our approach, cost of each consensus reaching process consists of two factors that can be subject to a weighted sum.

1. *Agents-related incurred cost*, related with changing their opinions towards the mutual agreement.
2. *Systems incurred cost*, so called "refusal cost" that results from the opportunity that each agent may accept or reject the advice given by the moderator.

An analysis of the change of opinion in the group decision making process in this paper is strictly linked to the model given in [3], guided by social and cognitive psychology theory of attitude change and small group dynamics [5]. The model shows how each agents' preference changes because of the interaction of some endogenous and exogenous forces. Nevertheless, the conceptualization of agent's opinion changing in response to forces is reflected in the mathematical formalism. A pivotal feature of the model is the presence of nonlinerity which fully reflects how the moderator brings strongly different testimonies of group member close enough to mutual consent.

2.1. Agents-related incurred cost

According to Gabbay's assumptions [3], each agent in the group has his own *natural preference*, which represents his worldview, beliefs and attitudes. Although *natural preference* is not a dynamical feature, it can be changed for the sake of fast shifts in natural testimonies. If an agent's preferences as to some pair of options is shifted from a natural preference because of a pressure of reaching a mutual consent, he will experience a psychological force that refuses this change, which Gabbay refers to as the *self-bias force*. In general, the further agent's preference is displaced away from his initial natural preference, the larger this force will be. In other words, the *self-bias force* increases in proportion to the divergence from the natural preference. Moreover, the strength of the *self-bias force* is determined by each agent's *commitment*; the more committed an agent is to his natural preference, the more difficult it becomes to move him away from it.

With respect to agents-related incurred cost, additional nonlinearity occurs. On the basis of facilitating role of the moderator, he should act in the spirit of behaviour change strategy. The moderator has to integrate losses (the loss function is concave upward). In theory, one lose of $3x$ is perceived as a less unpleasant than three separate losses of x [2]. In practice, when people think of something as one large loss or as a number of

smaller losses (losing $200 versus losing $100 twice), determining the situation as one large loss creates less negative approach than several smaller losses experienced separately [8].

2.2. Systems incurred cost

The second group of the cost boils down to the *influence force;* the force acting to change a given agent's preference (or several preferences) through the advice given by the moderator. The form of this force is illustrated in Figure 1 which plots the force on the agents, the *receiver*, due to the persuasive messages sent by the moderator, the *sender*. If the difference between receiver's preference and sender's expected preference is small enough, the *influence force* changes linearly with the discrepancy. However, when the influence force reaches its peak, the force increases asymptoting to zero for very high differences in preferences expectations. Noteworthly is a zone around the receiver's opinion, called the *area of acceptance*, for which the amount of changing in preferences increases as the differences in preferences increases because the messages sent by the moderator are experienced as equitable and reasonable. Beyond some critical difference, the amount of changes in preferences decreases with rising discrepancy. This zone is called *area of refusal* and all messages here are seen as irrational. Social judgment theory predicts a nonlinear dependence for the amount of change in preferences as a function of the message divergence, as presented in Figure 1.

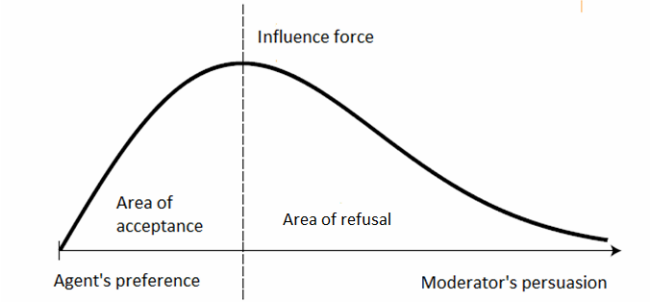

Figure 1. Influence force as a nonlinear function of distance between agent's preference and the preference suggested by the moderator [3].

Referring to the figure above, there are two parameters that determine the influence force: *area of acceptance* and the *strength* of the moderator on a particular agent. The *area of acceptance* scales the width of influence force curve as can be seen on Figure 1 and determines where maximum force is reached. However, the strength ranges the high of the curve, so that a larger strength value for moderator's persuasion on the agent implies a larger force. Contrary to *self-bias force* which stems from the structure of individual preferences, *influence force* arises from influence relationships.

2.3. Model summary

To stress the interesting insight into a group decision making, let us summarize the pivotal elements of the proposed model, concerning sociological and psychological

contours of agent's environment. First, each agent's preference change is subject to the following forces: the *self-bias force* and the *influence force*. The self-bias force is the pressure that each agent feels when his current preference as to some pair of options differs from his initial *natural preference*. Properties of the *self-bias force* are as follows: its value is proportional to the difference between agent's current preference and his natural testimony and it directly depends on the agent's *commitment* to his *natural preference*.

Apart from these endogenous factors, we also distinguish the *influence force*, i.e. the tension that an agent feels when he is persuaded by the moderator to change some of his preference. The *influence force* quantity increases approximately linearly for small preference differences but weakens for distances greater that the agent's area of acceptance. The force that the moderator exerts on each agent is based on the relationship and scaled by the *influence strength* which characterizes factors such as how often the moderator communicates with the agent or agent's perception of moderator's reliability.

3. Mathematical formalisation of the model

Let us assume a finite set of $n \geq 2$ options, $S = \{s_1, s_2, ..., s_n\}$ and a finite set of $m \geq 2$ agents, $E = \{e_1, e_2, ..., e_m\}$ who express their opinions as fuzzy preference relations.

1) The basic elements of the *agents-related incurred cost.*

Each agent e_k specifies his *natural preferences* as to the particular pairs of options in S. These testimonies are assumed to be an agent fuzzy preference relation R_k defined in $S \times S$ [8], characterized by its membership function:

$$\mu_{R_k} : S \times S \to [0,1], \tag{1}$$

such that the value $\mu_{R_k}(s_i, s_j) = r_{ij}^k$ is interpreted as a *natural preference* degree of option s_i over option s_j.

Commitment of each agent to his natural preference is defined as c_k, $c_k \in [0,1]$.

Self-bias force is determined as:

$$S_k = c_k (r_{ij}^{k\prime} - r_{ij}^k) \tag{2}$$

where: $r_{ij}^{k\prime}$ is the agents' e_k current opinion, cost of change agents's preference from r_{ij}^k to $r_{ij}^{k\prime}$ is some nonlinear function based on the distance in some metric (e.g. Manhattan distance). Nonlinearity of this function is stresses with respect to the rule of integration of losses. Namely, for each agent e_k, even a large change in his preferences

as to one specific pair of option s_i and s_j has a smaller cost than smaller changes in his preferences as to a few different pairs of options.

2) The critical elements for the *systems incurred cost.*

The *influence force* H_k, which is the force that the moderator impact on the agent e_k because of the difference between agent's preference and the preference proposed by the moderator.

The *strength* g_k, which denotes the strength of influence of the moderator of agent e_k because of their relationship, it ranges the height of the influence curve.

The *area of acceptance* l_k which defines the distance beyond which agent e_k start to refuse moderator's suggestions.

Again, relying on Gabbay's assumptions, the equation of influence force corresponding to the curve from Figure 1 is:

$$H_k = g_k \exp(\frac{(r_{ij}^{k\prime} - r_{ij}^{k})}{2l_k^2}) \tag{3}$$

The total cost of consensus reaching consist of two factors mentioned above can be subject to a weighted sum of S_k and H_k for all agents who had to change their preference in order to attain a sufficient degree of total agreement.

4. Conclusion and further research

In this paper we deal with the consensus reaching proceeds in a small group of agents who shows their preferences with respect to the set of options as fuzzy preferences. We proposed an approach based on the cost of reaching the consensus in the sense of evaluation of preference updating based on psychological reasons. We distinguish two types of forces representing by nonlinear functions either from the agent's point of view and his propensity of opinion change or from the position of the moderator whose proposition of preference updating may be accept or refuse by some group member.

We present some basic formalisms of the proposed model in order to provide a quantitative perception of some human manners. We do not consider here how to reach a consensus with a minimum cost, on the contrary we assume that the moderator has the unlimited budget during changing agents' preferences. The direction for the further research is to elaborate some algorithms where agents will not forced to change their mind too often or too many of their preferences. It should affect on their better collaboration and a better quality of the final decision.

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